# FORCED RESPONSE OF TURBINE BLADINGS WITH ALTERNATING MISTUNING AND FRICTION DAMPER COUPLING

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**Abstract:** Alternating mistuning of blades can help to avoid aerodynamic instabilities of turbine bladings. The influence of this type of intentional mistuning on the design of friction dampers has hardly been explored so far. To investigate the dynamics of alternately mistuned and nonlinearly coupled bladed disks a simple model is set up. The structure is represented by a plane lumped mass cyclic sector model with a single DOF oscillator for each blade. Underplatform dampers, referred to as friction dampers, are considered in terms of separate rigid bodies. Contact forces are evaluated by a penalty-based point contact model. Forced response functions and damper optimization curves are computed and show the potential of improved friction damping performance due to mistuning.

Keywords: alternating mistuning, intentional mistuning, friction damping, underplatform dampers

#### 1 Introduction

Turbine bladings are prone to mechanical vibrations mainly caused by fluid structure interaction. Alternating mistuning of blades can be used to avoid aerodynamic instability, see (Kaza & Kielb , 1982), (Kielb et. al., 2007), (Schönenborn et. al., 2012). However, this effect is restricted to frequency mistuning and alternating changes in blade profile or stagger angles may be adverse, cp. (Ekici et. al., 2008) and (Ekici et. al., 2013). Forced response of bladed disks with alternating mistuning has been found to increase the robustness with respect to random mistuning, (Hohl et. al., 2010), (Castanier & Pierre, 1998), (Castanier & Pierre, 2002). When aerodynamic coupling effects are included even a reduction of forced response levels due to mistuning is possible depending on the engine order of the excitation, (Kaza & Kielb, 1983), (Petrov, 2009), (Petrov, 2010). Similar results are found for frictionally coupled and mistuned systems, (Tatzko et. al., 2014). The positive effect of frequency mistuning on the damping performance could be observed in a numerical analysis of a real gas turbine stage with underplatform damper coupling, (Tatzko et. al., 2013). The present work focusses on bladed disks with alternating mistuning pattern and friction damper coupling which is hardly investigated to date. The mistuning is restricted to frequency mistuning of the blades only. A simple mechanical model of the structure is used to examine the fundamental dynamic behaviour that can lead to lower response amplitudes due to mistuning. Contact forces between friction dampers and blades are obtained by a penalty approach. The nonlinear system of equations is transformed into a set of complex algebraic equations. Forced response functions (FRF) are then computed using the single term harmonic balance method. Contact forces are evaluated in time domain with the AFT (alternating frequency time) method. Naturally, there are even more state-of-the-art contact and numerical models available (three-dimensional, Lagrangian contact formulation, multiharmonic approaches etc.). However, the objective of this paper is to clarify if an intentional mistuning is benefitial for the underplatform damper performance or not which can be demonstrated just as well by simplified models.

# 2 Mechanical model of the structure

The model is shown in figure 1 a). A cyclic sector with complex boundary conditions is used to represent the bladed disk. Each blade is modeled by a single DOF oscillator and each friction damper is described by a rigid body with two translational DOF in plane space. Similar simple models have been used to find essential parameter relations in friction damper design for turbomachinery, (Cameron et. al. , 1990), (Yang & Menq , 1998), and are still used to analyse dynamic effects in a wide parameter range, (Laxalde et. al. , 2007), (Murthy & Mignolet , 2013). The simplified structural modeling allows for analytical expressions that show the influence of certain parameters directly. However, many attributes of complex blade structures are not represented and results can generally not be used for quantitative predictions. Model refinement steps are inevitable. The chosen lumped model provides a basis for qualitative analyses with the following limitations. With one DOF only one mode shape of the blade can be approximated which must be far from other modes also referred to as isolated resonance. The nonlinear contact forces shall not affect this mode shape but can change the resonance frequency and damping.



Figure 1: Cyclic sector model with friction dampers (a) and point contact model with tangential and normal contact stiffnesses and coefficient of friction  $\mu$  (b).

The penalty springs must therefore not be interpreted as contact stiffnesses but residual stiffnesses. The equation of motion in time domain reads

$$M\ddot{\boldsymbol{u}} + D\dot{\boldsymbol{u}} + \boldsymbol{K}\boldsymbol{u} = \boldsymbol{F}_{\text{ex}} - \boldsymbol{F}_{\text{nl}}(\boldsymbol{u}, \dot{\boldsymbol{u}}), \tag{1}$$

with linear structural matrices M, D, K. For the sake of convenience the damping matrix is shown in the equation of motion but is actually considered in terms of modal damping, cp. equation 4. Alternating frequency mistuning is introduced by means of different spring stiffnesses of the two blades. External forces  $F_{ex}$  are harmonic and exhibit a certain phase shift  $\Delta \varphi$  between neighbouring blades defined by the engine order. The vector of displacements consists of the displacements of the blades and the translational motion of the dampers. The periodic response is approximated by its fundamental harmonic

$$\boldsymbol{u}(t) \approx \Re \left\{ \hat{\boldsymbol{u}} \mathrm{e}^{\mathrm{i}\Omega t} \right\}.$$
<sup>(2)</sup>

After inserting the approximation the equation of motion is balanced using a Galerkin projection which yields a complex algebraic set of equations for the harmonic coefficients

$$\hat{\boldsymbol{U}} = \hat{\boldsymbol{H}} \left( \hat{\boldsymbol{F}}_{\text{ex}} - \hat{\boldsymbol{F}}_{\text{nl}} \right) ; \ \hat{\boldsymbol{H}} = \text{blkdiag}(\hat{\boldsymbol{H}}_{\text{b}}, \hat{\boldsymbol{H}}_{\text{d}})$$
(3)

$$\hat{\boldsymbol{H}}_{\rm b} = \sum_{m=1}^{2} \frac{\boldsymbol{\phi}_{m} \boldsymbol{\phi}_{m}^{\rm H}}{\omega_{m}^{2} - \Omega^{2} + \mathrm{i}2D\omega_{m}\Omega} ; \; \hat{\boldsymbol{H}}_{\rm d} = \mathrm{diag}(-1/m_{\rm d}\Omega^{2}). \tag{4}$$

The dynamic compliance matrix  $\hat{H}_{\rm b}$  of the blades is obtained by the dyadic products of the eigenvectors  $\phi_m$  and modal damping values D for each modeshape. The nonlinear contact force amplitudes  $\hat{F}_{\rm nl}$  are evaluated with the point contact model shown in figure 1 b) using the AFT method proposed by (Cameron & Griffin , 1989). The harmonic coefficients are obtained using the FFT of the signal. A Newton type procedure is then used to solve the algebraic system of equations for the amplitudes of the blades and the dampers.

# **3** Forced response results

In figure 2 the FRF results are plotted normalized by the resonance amplitude of the uncoupled (free) and tuned system. The tuned response for an engine order EO = 1 is plotted in figure 2 a) and shows a typical damper optimization curve with a unique response minimum. Figure 2 b) shows the envelope of the response of the mistuned blades without relative coupling ( $k_{rel} = 0$ ). The damper optimization curve exhibits two local minima and in part lower response levels. (Kaza & Kielb , 1983) found similar results for aerodynamically coupled systems. They observed reduced response amplitudes due to alternating mistuning for low values of the engine order EO. With increasing centrifugal force the response in figure 2 b) approaches the tuned results. The damper contact forces reduce the mistuning effect due to a relative coupling. This effect is similar to a linear relative coupling with coefficient  $k_{rel}$  which can be interpreted as a compliance of the disk. A modal analysis of the linear structure reveals the adverse influence of a relative coupling on alternating frequency mistuning. The spring stiffnesses are described by dimensionless parameters  $\epsilon_{mist}$  and  $\epsilon_{rel}$ . Mistuning is defined by  $k_A = (1 - \epsilon_{mist})k$  and  $k_B = (1 + \epsilon_{mist})k$  for the alternating A-B blade pattern and the relative coupling reads  $k_{rel} = \epsilon_{rel}k$ . The complex eigenvectors of the two blade sector can be obtained analytically in travelling wave form as follows

$$\boldsymbol{\phi}_1 = \begin{bmatrix} 1\\ \alpha e^{+i\Delta\tilde{\varphi}} \end{bmatrix} ; \quad \boldsymbol{\phi}_2 = \begin{bmatrix} -\alpha e^{-i\Delta\tilde{\varphi}}\\ 1 \end{bmatrix}. \tag{5}$$



Figure 2: FRF results of the tuned (a) and the alternately mistuned assembly (b).

A certain mode shape of the bladed disk is related to a phase shift  $\Delta \tilde{\varphi}$  between neighbouring blades. The amplitudes are affected by the mistuning and the factor  $\alpha$  reads

$$\alpha(\epsilon_{\rm mist}, \epsilon_{\rm rel}, \Delta\tilde{\varphi}) = \frac{\sqrt{\left(\frac{\epsilon_{\rm mist}}{\epsilon_{\rm rel}}\right)^2 + 4\cos^2\Delta\tilde{\varphi} - \frac{\epsilon_{\rm mist}}{\epsilon_{\rm rel}}}{2\cos\Delta\tilde{\varphi}}.$$
(6)

The mistuning factor  $\alpha$  shows directly that a relative coupling reduces the frequency mistuning of the blades. In the tuned case only one complex eigenvector contributes to the response. If the system is mistuned, orthogonality is disturbed and both eigenvectors are excitable by one engine order.

Concluding, alternating frequency mistuning causes a mode coupling of two complex mode shapes of the bladed disk. The overall damping performance of underplatform dampers can be improved depending on the relative motion of the additionally excited eigenmode or the engine order of the excitation, respectively. However, any relative coupling causes a reduction of the alternating mistuning level and possible benefitial effects on the friction damping performance are reduced as well.

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