PARAMETRIC ANALYSIS OF THE NONLINEAR BEHAVIOR OF ROTATING STRUCTURES

L. Xie[1,2,3], S. Baguet[1], B. Prabel[2,3] and R. Dufour[1]

[1] Université de Lyon, CNRS, INSA-Lyon, LaMCoS, UMR5259, 69621 Villeurbanne, France
E-mail: *sebastien.baguet@insa-lyon.fr*, *regis.dufour@insa-lyon.fr*
[2] CEA-Saclay, DEN,DANS,DM2S,SEMT,DYN, 91191 Gif-sur-Yvette, France
E-mail: *lihan.xie@insa-lyon.fr*, *benoit.prabel@cea.fr*
[3] IMSIA, UMR 9219 CNRS-EDF-CEA-ENSTA, Université Paris Saclay, 91762 Palaiseau Cedex, France

Abstract: An efficient frequency-domain method is presented for the rapid parametric analysis of stability changes in nonlinear rotating systems which are modeled by three-dimensional finite elements. This method provides directly the stability boundary with respect to parameters such as the system nonlinearity or excitation level. Firstly, the response curve is calculated by combining Harmonic Balance Method and continuation. Then stability of equilibrium solutions is determined thanks to Lyapunov exponents. The singular points where a stability change often arises are detected with the sign change of the Jacobian determinant and then located through a penalty method that increases the solving equation system by a completing constraint. Tracking these points, which provides an efficient way to analyze parametrically the nonlinear behavior of a system, can be fulfilled, once again, by the continuation technique.
Keywords: *non-linear rotordynamics, parametric analysis, stability boundary, balance harmonic method, bifurcation*

1 Numerical methods

Basic rotordynamics modeling is a classical and efficient way to design and predictive maintenance in most industrial applications. However, when we have to take into consideration of non-linearity, such as a crack, rotor-stator contact, hydrodynamic bearing, etc, a three-dimensional finite element model will define more precisely the geometry and the interaction of system. Due to the large number of DOFs and the broad range of study frequency, the computation time can be quite prohibitive. The Harmonic Balance Method (HBM) is thus employed due to its efficiency in predicting steady state behavior. The nonlinear differential equation is transformed into a nonlinear algebraic equation system :

$$\mathbf{R}(\mathbf{X},\omega) = \mathbf{Z}(\omega)\mathbf{X} + \mathbf{F}_{NL}(\mathbf{X}) - \mathbf{P}(\omega) = 0 \quad (1)$$

where $\mathbf{Z} = diag(\mathbf{K}, \mathbf{Z}_1, ..\mathbf{Z}_k, ..\mathbf{Z}_N)$ with $\mathbf{Z}_k = \begin{bmatrix} \mathbf{K} - k^2\omega^2\mathbf{M} & \omega\mathbf{C} \\ -\omega\mathbf{C} & \mathbf{K} - k^2\omega^2\mathbf{M} \end{bmatrix}$,$\boldsymbol{M}, \boldsymbol{C}, \boldsymbol{K}$ are generalized mass, damping and gyroscopic, and stiffness matrices of finite element model, ω is the excitation frequency, $\mathbf{F}_{NL}$ and $\mathbf{P}$ are vectors of harmonic coefficients for nonlinear forces and excitation forces. With the help of continuation method, all dynamic equilibrium solutions of nonlinear systems are determined.

Then, determining the local stability of a periodic solution is particularly interesting in an engineering context since only stable solutions are experimentally encountered. Moreover, a change in the stability can lead to significant, qualitative, and possibly dramatic changes in the system response. Therefore, Lyapunov exponents which are eigenvalues of Jacobian are sought for stability analysis of periodic solutions.

In order to assess the influence of parameters on the dynamic behavior in a more economic way, the direct parametric analysis is necessary for numerical investigation of nonlinear systems. Singular points which include the limit (turning) points (LP), branch points (BP), Neimark-Sacker bifurcation points(NS), are often accompanied by a change of stability. Here, the Neimar-Sacker bifurcation points are Hopf bifurcation points for the algebraic system (1).

By definition, a limit point appears when the Jacobian possesses a zero eigenvalue. The determinant of the Jacobian matrix is thus monitored since its sign change indicates whether a limit point or a bifurcation point has emerged. Then, if the determinant of augmented Jacobian (includes continuation) is nil, a branch point (BP) has appeared, otherwise, it is a limit point (LP). In the mean time, a Neimark-Sacker bifurcation point occurs when a pair of complex conjugate eigenvalues crosses the imaginary axis of the complex plane. The singular points can be located by adding a new constraint equation which characterizes the points to the solving system.

Next, applying once again the continuation method to the augmented system brings about direct tracking of singular points as a function of nonlinear parameters or excitation level. Thus, parametric analysis of the nonlinear behavior of a dynamical system is achieved, the stability boundary or regime change boundary is directly determined.

2 Numerical applications - Nonlinear Jeffcott rotor

The first test case is a modified Jeffcott rotor which can come into contact with a stator that is modeled as an added stiffness [Jiang (2009)]. The equations of motion are the following

$$
\begin{aligned}
m\ddot{x} + c\dot{x} + kx + k_c(1 - \frac{h}{r})(x - \mu y sign(v_{rel})) = p_b\omega^2 cos\omega t \\
m\ddot{y} + c\dot{y} + ky + k_c(1 - \frac{h}{r})(\mu x sign(v_{rel}) + y) = p_b\omega^2 cos\omega t
\end{aligned}
\tag{2}
$$

where k_c is the stiffness of contact surface, h is the clearance between the rotor and the stator, $r = \sqrt{x^2 + y^2}$ is the radial displacement, p_b is the unbalance amplitude and v_{rel} is the relative velocity between the rotor and the stator at the contact point. When $r < 0$, there is no rub between the rotor and the stator, $k_c = 0$.

Assessments are carried out with the friction coefficient μ as the varied parameter. The eigenvalues of Jacobian help to locate the singular points by their characteristic (limit point and Neimark-Sacker point as shown in Fig.1) while the stability assessment gives information for stable and unstable solutions (solid line and dotted line). The calculation in Fig.1 is carried out with a friction coefficient $\mu = 0.2$. Before the dimensionless excitation frequency reaches 0.154, there is no contact between the rotor and the stator. Then synchronous full annular rub occurs when the vibration exceeds the rotor-stator initial clearance. Next, when the frequency is beyond 0.289, quasi-periodic partial rub is the only stable motion of the rotor. The intersection point is a Neimark-Sacker bifurcation point. Several forced response curves for different friction coefficient values are calculated and plotted in 3D as shown in Fig.2. As observed, the Neimark-Sacker points have marked the motion change (from periodic to quasi-periodic motion), while the limit points distinguishes the stability change (on the range of $\mu < 0.11$).

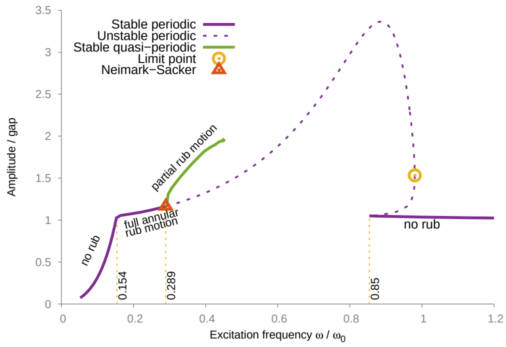

Figure 1: Forced response curve of Jeffcott rotor for $\mu = 0.2$

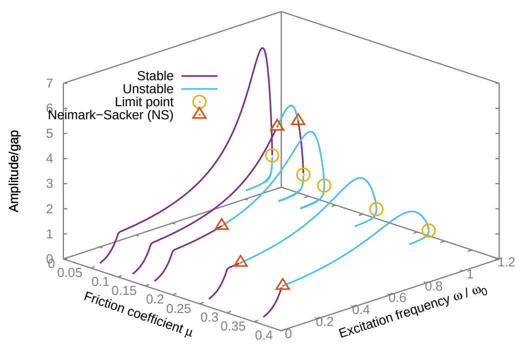

Figure 2: Forced response curve of Jeffcott rotor for several friction coefficient values

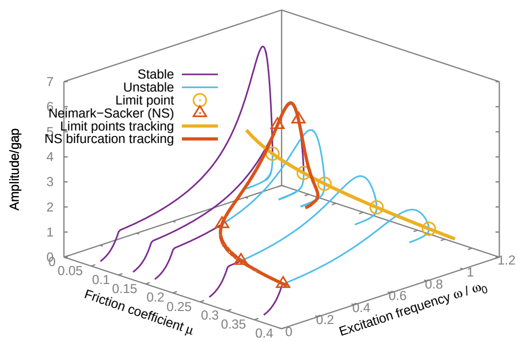

Figure 3: Limit point tracking and Neimark-Sacker point tracking of Jeffcott rotor as a function of friction coefficient μ

The method presented here calculates at first the forced response for a fixed μ, once a singular point is detected and located, the continuation technique with the varying parameter μ is added to the solving system so that all the limit points (or the Neimark-Sacker points) are determined directly. The tracking costs as little as one forced response calculation. The example here has demonstrated the efficiency of the presented method for behavior analysis of nonlinear rotating system.

Numerical developments are fulfilled in both Matlab and Cast3m [CAST3M (2014)], paving the way for application of the method to the nonlinear dynamics of rotors modelized with 3D finite elements.

References

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